week 5 — Metrics | 4 forces & unification

Metrics

Minkowski space
- Flat iff slow, no mass
  - Not very practical / interesting in that regard
- what is cool:
  - EVERYTHING is locally minkowski (without gravitational singularities)

What about that caveat? (gravitational singularities)
- Guess ⇒ (black holes)

Black holes ↔ Schwarzschild metric
- 1st exact solution to Einsteins equations
- developed while on the Russian front WWII

SHOW LATER ↓

$$c^2 d\tau^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)$$

$r_s = \frac{2GM}{c^2}$

special case of Pythagorean $(\Delta d)^2 = (\Delta a)^2 + (\Delta b)^2$

What are some special things about a black hole?

Among others: event horizon

Derive schwarzschild radius, r_s

$\left(1 - \frac{2GM}{r c^2}\right)$ — solve for 0

$1 = \frac{2GM}{r c^2}$

$r = \frac{2GM}{c^2}$ — Now let's get a feel for that

$$R_s = \frac{2GM}{c^2}$$

Schwarzschild Radius

1st – What does it mean?

- Any mass apparently works
  - must be inside r_s
    - hence we are not black holes
- More mass → larger r_s

$G = 6.67 \cdot 10^{-11}$

$c = 3 \cdot 10^8$

Practice

	Mass (kg)	R_s (m)
Milky Way Black Hole	$8.2 \cdot 10^{36}$ ($4 \cdot 10^6 M_\odot$)	$1.2 \cdot 10^{10}$
Earth	$6 \cdot 10^{24}$ kg	$8.9 \cdot 10^{-3}$
You	~ 70 kg	$1.04 \cdot 10^{-25}$

Planck length: $1.6 \cdot 10^{-35}$

Proton radius $.87 \cdot 10^{-15}$

cool pictures of Sagitarius A (Milky way supermassive black hole) on wikipedia

$R_\oplus = 6370$

IF THERE IS TIME

Redshift

$$\frac{\left(1 - \frac{2GM}{r_1 c^2}\right)}{\left(1 - \frac{2GM}{r_2 c^2}\right)} = 1 + z = \frac{\nu}{\nu_0}$$

de Sitter Space

massless

positive cosmological constant

expansion → redshifts

Anti-de Sitter space (AdS)

massless

negative cosmological constant

contraction → blueshift

Doesn't correspond to any observed system but it is promising for exploring quantum Gravity