Week 2

Last week: Expanding space? — Just a nice philosophical musing?

This week: How can that be true??
earlier – how describe how things expand

Now: What makes it expand — Always: why?

~~19~~ 1922 – Friedmann (Russian)
- Started with energy conservation
  - kept following the equations
  - it ~~fit~~ perfectly fit into Einstein's equations
  * because ~~it started~~ ~~m~~ it required every step to be valid

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$\rho = \frac{M}{V}$ density

$$\left(\frac{\ddot{a}}{a} = -\frac{4}{3}G\left(\rho + 3\frac{P}{c^2}\right)\right)$$

$k = -1$ open
0 closed
1 flat

Note – derivitive
$$\dot{a} = \frac{\Delta a}{\Delta t} = \frac{\text{change in } a}{\text{change in } t}$$

Let's play with it
- Question: What did we say the geometry was? open / flat ← approx. / closed

So k is? ~ 0

Given $H_0 = 2 \cdot 10^{-18}\ \frac{1}{s}$
$G = 6.67 \cdot 10^{-11}$

Only do if they have a chem background
- not worth teaching the conversions

find $\rho_0 = 10^{-26}\ \frac{kg}{m^3}$ Corresponds to $\frac{\#\ H\ \text{molecules}}{m^3} \sim 6\ \text{atoms}/m^3$
$N_A = 6.022 \cdot 10^{-23}$
10^{25} for air @ STP

Still Friedmann Equations

Fun with ρ (rho - mass density)

Critical Mass Density, ρ_c

Def = the mass density corresponding to a perfectly flat universe
- it's the ρ_c we just found!

how to motivate pointing out these 2 quantities?

But what if it's a little too dense?

$$\rho_c = \frac{3H^2}{8\pi G}$$

Omega - $\Omega \equiv \frac{\rho}{\rho_c}$

$\Omega > 1$ - too massive - closed universe - collapse
$\Omega = 1$ - perfectly flat - what we see
$\Omega < 1$ - too light - open universe

Analogy : Gravity & Escape Velocity

An object can escape earth's gravitational force, if moving fast enough. If the earth had more mass, the object feels pulled back more - so it needs to go faster

Earth $\longrightarrow$ Universe
- it's mass $\longrightarrow$ ρ - mass density
faster velocity $\longrightarrow$ faster expansion $H(t) = \frac{\dot{a}}{a}$

Friedmann Eq'ns (continued)

Expansion → Change in mass density

Normal Matter → If you have a 1 kg block in 1 m^3 of otherwise vacuum, and you double box size (size ≡ length - 1 dimension) then you now have $1\,kg / (2m)^3 = 1/8\ m^3$

ρ scales with $[length]^3 \rightarrow \frac{1}{a^3}$

Relativistic Matter → [Requires chem knowledge]

Similarly to normal matter, we find # particles $\propto \frac{1}{a^3}$

But by redshift ($E \propto \frac{1}{\lambda} \propto \frac{1}{a}$)

$\rho_{rel} \propto E_{density} = \frac{\#\ photons}{volume} \cdot \frac{Energy}{volume} \propto \frac{1}{a^3} \cdot \frac{1}{a}$

$\rho_{normal} \propto \frac{1}{a^3}$ $\qquad\qquad$ $\rho_{rel} \propto \frac{1}{a^4}$

NOTE: Only do this topic with plenty of time. It would probably be best to save it to be a dedicated lesson.

Would it be confusing to teach $\rho \propto \frac{1}{a^3}$ - just to teach them other rules later?

$\rho_{rel} \propto \frac{1}{a^4}$ $\qquad$ $\rho_{dark\ energy} \propto 1$